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PROBLEMS IN THE THEORY OF PROBABILITY AND MATHEMATICAL STATISTICS

by

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PROBLEMS IN THE THEORY OF PROBABILITY AND MATHEMATICAL STATISTICS

A. N. Kolmogorov

My survey of the modern state and basic directions of the development of research in the theory of probability and mathematical statistics I would like to begin by calling attention to the issue of the 4th volume of the collection of writings of S. N. Bernshteyn (the volume contains all his work on the theory of probability and its applications). If something in this publication now has interest, at least because of the advantage for the history of science, since in more acceptable form it has gone into textbooks, then very important here is the store of ideas, which still by anything but in exhaustive fashion, and sometimes in insufficient manner, have become known to young investigators.

The problematic in the field of limiting theorems of the theory of probability, which has occurred to P. L. Chebyshev and A. M. Lyapnov, and which has received substantial development in the works of A. A. Morkov and S. A. Bernshteyn in the direction of the study of dependent values, seemed at one time to be inexhausible, but now is passing through a period of flowering. To problems which have some bearing in this connection there was devoted the report of V. A. Statulyavichus (Russian transliteration), "Limiting theorems in boundary problems and some of their applications",

which was read at the General Convention of the Departments of Mathematics of the Academy of Sciences on October 29, 1966, dealing with the theory of probability and mathematical statistics.

At the same meeting A. A. Vorovkov read a report about the cycle of papers which followed another current in the field of limiting theorems, begun, apparently, by the Swedish mathematician G. Kramer, towards the so-called "Theorem of large deviations." It is worthwhile to go more into detail about the applied significance of these papers in mathematical statistics, outlined by A. A. Vorovkov. In the simplest typical problems of mathematical statistics there are two parameters - "level of significance" α (allowable probability of erroneous judgment) and the number of observations n. The approach supported by the limiting theorems of Chebyshev's type corresponds to limiting transition with constant α and n approaching infinity. However, in practice n remains often only of the order of some hundreds and even tens, and the level of significance ordinarily is chosen from 0.05 to 0.001. It is probable that there will be more and more problems requiring assurance of high "reliability", that is, very small values of α. Therefore, often the formulas of the "theory of large deviations" which relate to the asymptotic with α approaching zero turn out to be more applicable.

A. A. Markov made the start in the study of the broad class of random processes now called everywhere the "Markovian" processes. In the development in this direction our country also continued to play a very big role, in recent years mostly through the works of the school of Ye. B. Dynkin. It seems to me that in the theory of the Markovian processes, although there are problems still remaining there of the type of getting as general conditions as possible of the applicability of the basic theorems; freeing the theorems of excess prerequisites, now more essential are the searches for new problematic, embracing even without refinement of the mathematical apparatus, a broader circle

of applications. In particular the study of Markovian processes which are only partially observable is very timely, i.e., the processes of the kind

$$x(t) = \{x_1(t), x_2(t)\},$$

where only the component $x_1(t)$ is observable. Extremely interesting ideas about the ways of solving the problems cropping up here are formulated by R. L. Stratonovich sometimes no longer do not only not possess mathematical refinement, but often are carried along on a level which does not assure that "reasonable strictness which without being absolute guarantees against errors" (expression of A. N. Krylov). In the report by A. D. Venttsel' it was told how some part of the theory of "conditional Markovian processes" can be constructed without due strictness.

There is being intensely developed the spectra theory of the stationary random processes the strict bases of which were laid in our country by A. Ya. Khinchin. Now the main attention, perhaps under the influence of the ideas of N. Viner, is here given to the attempts to construct a spectral "nonlinear" theory. This is very essential since the thinking of the specialists in the field of radio technology, transmission of information, etc, runs along the line of the use of spectral presentation, but in continuous spectra, typical for the random processes in essence there remains mathematically developed only the linear theory, in many practically important applications quite insufficient.

In the field of the theory of information our scientists have overtaken foreign science. One can consider now that the lag has been made up for, and the work of the deceased A. Ya. Khinchin and that of the representative of our young generation R. L. Lobrushin have taken already an outstanding position in international science.

Information, by its nature, is not a very specially probable concept. The original presentation of information as a number of binary digits necessary for the separation of a determined object from a finite set of objects contains nothing at all in common

with the theory of probability. Only in higher departments of the theory of information do there dominate probability methods. It is possible, however, that the relationship between the theory of information and the theory of probability will change radically. I do not wish now to develop here the concept (I find it personally more and more attractive) accordance with which these relationships can be reverse contemporaries and not that the theory of probability will be the basis of higher departments of the theory of information, but that in the basis of the theory of probability there will lie the concepts of the theory of information.

I note here only the rise of a new branch of the "theory of dynamic systems", i.e., the general theory of nonrandoms or strictly determined processes, in which the concepts of the theory of information (beginning with the information concept of "entropy") play the basic role. The great analogies between the dynamic systems with the property of "intermingling" and the random processes have been understood or a long time. But now in the works begun by myself and continued by V. A. Rokhlin, and especially by Ya. G. Sinay these analogies have been considerably extended. In particular Ya. G. Sinay in broad assumptions and for some completely classic model systems (elastic balls in a box) proved the long-established hypothesis of the asymptotically normal distribution of the "times of remaining" in the different regions of phase space. Apparently for the classical dynamic systems determinable by vector fields on compact diversities there are two extreme cases - the case studied by V. I. Arnold and myself of "near periodicity" and the case of the "K systems" with intermingling prove to be in some sense basic.

In the field of mathematical statistics, notwithstanding the fact that in the schools of N. V. Smirnov and Yu. V. Linnik many brilliant investigations have been carried out, the adtivity of the Soviet mathematicians is far from sufficient. Apparently such a situation is brought about by the fact that the development of mathematical

statistics is closely connected with the direct work with actual statistical material. However, for qualified Soviet mathematicians contact with such a kind of work on actual material still remains, if not rare, at least incidental and somewhat random. As to the their outstanding attainments in the solution of difficult analytical problems which arise in mathematical statistics this was reported on by Yu. V. Linnik. In the V. A. Steklov Mathematical Institute of the Academy of Sciences of the USSR under the direction of N. V. Smirnov and L. N. Bol'shev there is extensive work being done in the issuing of mathematical tables needed for statistical practice and the computing of a series of new tables.

Some groups of mathematicians in Moscow, Leningrad, and other cities with enthusiasm are helping the scientists of other specialities in solving practical problems (in the field of biology, geology, etc.) by statistical methods. But above one has already spoken about the somewhat random, uncoordinated, and sometimes also dilettante character of this work. In the future the Department should devote its attention to the question of a more rational and wider organization of that work at one of its coming sessions.

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ABSTRACT: This is a survey of research in the theory of probability. The works of several scientists in this field are cited. The "Markovian" process is discussed. The application of mathematics to the different scientific fields is cited.

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